Derivation of kinetic energy formula

Consider a mass m accelerated from rest by a force F. When the object's displacement is s, its velocity is v.

The energy transferred to the mass by the force is equal to the work done. The equation for work done is:

work done = force x distance

$= F \times s$ (1)

Newton's second law of motion states that $F = ma$.

We also know that:

$v^2 = u^2 + 2as$

The mass accelerates from rest, so $u = 0$. Therefore:

$$s = \frac{v^2}{2a}$$

Substituting for F x s in (1), we find that:

$$\text{work done} = ma \times \frac{v^2}{2a} = \tfrac{1}{2}mv^2$$

The work done is equal to the energy gained by the mass as it speeds up, so we can see that the object's kinetic energy must be equal to $\tfrac{1}{2}mv^2$. This gives us the equation:

$E_k = \tfrac{1}{2}mv^2$